ISSN: 2321 – 7537
IJEDST (2014), 2(1):1-18

Impact of Bhatnagar's First & Second Alternative Formulations on Rankings of the Countries in UNDP's Human Development Reports during 1995 to 1998

Ravi Kant Bhatnagar[1] and Neeraj Tiwari [2]
1 Additional Director General (Market Research), Ministry of Tourism (Government of India), Dalhoussie Road, New Delhi, INDIA.
2 Professor & Head in Department of Statistics, Kumaun University, S.S.J.Campus, Almora, Uttarakhand, INDIA.
bhatnagar.rk2010@gmail.com

Abstract:
Bhatnagar [1&2] proposed his first and second alternative formulations as viable substitutes for faulty Atkinson-based multi-step formulation of utility function used in UNDP's Human Development Reports till 1998. In this paper we study impacts of both the formulations on rankings of the countries in Human Development Reports during 1995 to 1998, which happened to be the period of stabilized UNDP's methodology developed on the premise of multi-step utility functions after initial nascent phases of evolution of the Human Development Index.
Keywords: Human Development Index, Income-transforming Utility function

INTRODUCTION

While aiming at human-centred development through widening of people's options to lead a long and healthy life, to be knowledgeable, and to maintain a descent standard of living, it needs to be appreciated that there is no automatic link between the level of per capita GDP in a country and the level of its human development. In the wake of limitations of income alone as a comprehensive yardstick for measurement of human development, people therefore, started looking beyond figures of growth rate in GDP and emergence of composite indices of well-beings and standard of living index began to appear initially as Physical Quality of Life Index (PQLI) and Basic Needs Approach etc. before the advent of concept of Human Development Index (HDI) in early Nineties. Co-terminus with the growth rate in GDP, the quality of life thus also began to be regarded as an important aspect of human development. Keeping in view longevity, education and income as the most important capabilities affecting wellbeing of the people at large, the UNDP in its HDRs accordingly dogmatized adoption of life expectancy at birth, educational attainment and real per capita GDP income in PPP$ for the purpose of framing the HDI as the simple (un-weighted) average of the values of measurement indicators for life expectancy, educational attainment and the adjusted per capita GDP. Different countries are ranked according to the values of their HDI. The basic variables included in the Human Development Reports (HDRs) of UNDP for computation of values of HDI for different countries were namely, the life expectancy at birth, adult literacy rate, combined gross enrolment ratios and the real per capita GDP income. While the measurement indicator for life expectancy for different countries in the HDRs is computed directly from their data on the life expectancy at birth, the values for the measurement indicator on educational attainment for the countries are arrived at as a synthetic combination of the indices of the adult literacy rate and the combined enrolment ratios in the proportion of 2/3 and 1/3. The computation for measurement indicator on the third component namely, real per capita GDP income has undergone drastic changes over time. Initially a truncated logarithmic function of per capita GDP income was considered by UNDP in 1990 for 'adjusting' the actual values for different countries, followed by adoption of multi-step formulation of income-transforming-utility function coupled with the concept of threshold income level during 1991 to 1998 and finally again reverting back from 1999 to logarithmic utility function but as a non-truncated one as against the truncated logarithmic utility function besides abandoning the concept of threshold income level. The precise 'adjustment' needed during 1991 to 1998 for any real per capita GDP income depended upon its gap from the threshold income level with the stipulation that 'adjusting' was not called for if the real per capita GDP income was below the threshold income level. Alternative formulations under the premise of multi-step utility functions have also been proposed in the literature and the aim of the present paper is to discuss their implications on the rankings of different countries.

Considering the fragmentation of the entire range of real per capita GDP for different countries in any HDR into income-bands as 0 to y^*, y^* to $2y^*$, $2y^*$ to $4y^*$, $4y^*$ to $6y^*$, $6y^*$ to $8y^*$ and so on the basis of threshold income level (say, y^*), Bhatnagar's First Alternative Formulation (BFAF) has been proposed in literature (See Equations (16A) to (16F) in [1], p.56), as a viable substitute for erroneous Atkinson based formulation of multi-step utility function. Denoting the utility function for various income bands through appropriate subscripts which actually correspond to coefficients of y^* in the terms specifying the start of the income bands, we can re-write Bhatnagar's First Alternative Formulation (BFAF) as under:

$W_0 = y$, for $0 \le y \le y^*$ (1A)

$W_1 = y^* + y^* \log(y/y^*)$, for $y^* < y \le 2y^*$ (1B)

$W_2 = y^* + (1/2)y^*(\log 2) + (1/2)y^* \log(y/y^*)$, for $2y^* < y \le 4y^*$ (1C)

$W_4 = y^* + (1/4)y^*(2\log 2 + \log 4) + (1/4)y^* \log(y/y^*)$, for $4y^* < y \le 6y^*$ (1D)

$W_6 = y^* + (1/8)y^*(4\log 2 + 2\log 4 + \log 6) + (1/8)y^* \log(y/y^*)$, for $6y^* < y \le 8y^*$ (1E)

ISSN: 2321 – 7537
IJEDST (2014), 2(1):1-18

$W_{2n} = (\sum^{n}_{m=0} (1/2^m)\ y^* \log(2m)) + (1/2^n)y^* \log(y/y^*)$, for $2ny^* < y \leq 2(n+1)y^*$ and $n \geq 1$ (1F)

It may be observed first two income bands namely 0 to y^* and y^* to $2y^*$ are of same width y^* but the subsequent income bands though being of same width, have double the span of the first two intervals. However, by keeping the width of all income bands formed on the basis of the threshold income as same, Bhatnagar's Second Alternative Formulation (BSAF) appears in the literature (See Equations (7A) to (7H) in Bhatnagar's i.b.i.d [2], p.135-136) as a viable substitute for the Atkinson-based multi-step utility function. Following the analogous notations as earlier, Bhatnagar's Second Alternative Formulation (BSAF) can be re-written as:

$U_0 = y$, for $0 \leq y \leq y^*$ (2A)

$U_1 = y^* + (1/2)y^* \log(y/y^*)$, for $y^* < y \leq 2y^*$ (2B)

$U_2 = y^* + (1/6)y^* (\log 2) + (1/3)y^* \log(y/y^*)$, for $2y^* < y \leq 3y^*$ (2C)

$U_3 = y^* + (1/6)y^*(\log 2) + (1/12)y^*(\log 3) + (1/4)y^*\log(y/y^*)$, for $3y^* < y \leq 4y^*$ (2D)

$U_4 = y^* + (1/6)y^*(\log 2) + (1/12)y^*(\log 3) + (1/20)y^*(\log 4) + (1/5)y^*\log(y/y^*)$, for $4y^* < y \leq 5y^*$ (2E)

$U_5 = y^* + (1/6)y^*(\log 2) + (1/12)y^*(\log 3) + (1/20)y^*(\log 4) + (1/30)y^*(\log 5) + (1/6)y^*\log(y/y^*)$, for $5y^* < y \leq 6y^*$ (2F)

$U_{n-1} = y^* + (1/6)y^*(\log 2) + (1/12)y^*(\log 3) + (1/20)y^*(\log 4) + (1/30)y^*(\log 5) + \ldots\ldots + (1/(n-1)n)y^*(\log(n-1)) + (1/n)y^*\log(y/y^*)$, for $(n-1)y^* < y \leq ny^*$ (2G)

$U_n = y^* + (1/6)y^*(\log 2) + (1/12)y^*(\log 3) + (1/20)y^*(\log 4) + (1/30)y^*(\log 5) + \ldots\ldots + (1/(n(n+1)))y^*(\log n) + (1/(n+1))y^*\log(y/y^*)$, for $ny^* < y \leq (n+1)y^*$ (2H)

The above general Equation (2H) can compactly be written as

$U_n = y^* + (\sum^{n}_{m=1} \{1/(m(m+1))\}y^*\log(m)) + \{1/(n+1)\}y^*\log(y/y^*)$, for $ny^* < y \leq (n+1)y^*$ (2I)

We now concentrate more closely on Bhatnagar's First and Second Alternative Formulations of utility function and study their impacts, under different scenarios, on the rankings of various countries in the HDRs from 1995 to 1998 during which UNDP's methodology with regard to computation of the HDI remained stabilized.

EFFECT ON THE RANKS OF COUNTRIES DUE TO REPLACEMENT OF UNDP's ATKINSON BASED MULTI-STEP UTILITY FUNCTION WITH BHATNAGAR's FIRST & SECOND ALTERNATIVE FORMULATIONS WHILE RETAING SAME LEVELS OF THRESHOLD INCOME IN CORRESPONDING HDRs

Using the basic data of HDRs from 1995 to 1998, we have computed the variations in the ranks of all the countries, due to Bhatnagar's First and Second Alternative Formulations of the utility function. Let us begin by first observing the changes in the rankings of countries belonging to the Indian Sub-continent as follows.

Table 1. Effect on the ranks of countries of Indian Sub-Continent due to Bhatnagar's First and Second Alternative Formulations retaining corresponding threshold income levels in the HDRs

Utility Function's formulation	Country	HDR1995		HDR1996		HDR1997		HDR1998	
		Old rank	New rank	Old rank	New rank	Old rank	New rank	Old rank	New rank
Bhatnagar's First Alternative Formulation	Bangladesh	146	147	143	143	144	147	147	147
	Bhutan	160	160	159	159	155	156	155	157
	India	134	135	135	134	138	137	139	137
	Nepal	151	151	151	152	154	154	152	153
	Pakistan	128	133	134	135	139	139	138	139
Bhatnagar's Second Alternative Formulation	Bangladesh	146	148	143	143	144	144	147	147
	Bhutan	160	160	159	159	155	156	155	155
	India	134	134	135	134	138	138	139	138
	Nepal	151	151	151	152	154	154	152	153
	Pakistan	128	131	134	135	139	139	138	139

Table 2. Variations in ranks of prominently affected countries in HDR 1998 on replacing utility function from Atkinson based to Bhatnagar's First Alternative Formulation retaining corresponding threshold income levels in the HDRs

Countries	Effect on Ranks due Bhatnagar's First Alternative Formulation	Countries	Effect on Ranks due Bhatnagar's First Alternative Formulation
Kuwait	19up	Costa Rica	13down
Qatar	17up	Botswana	12down
Mauritius	17up	Spain	9down
UAE	11up	Antigua & Barbuda	9down
Bahrain	10up	Fiji	8down
Malaysia	10up	Dominica	8down

Table 3. Variations in ranks of prominently affected countries in HDR 1997 on replacing utility function from Atkinson based to Bhatnagar's First Alternative Formulation retaining corresponding threshold income levels in the HDRs

Countries	Effect on Ranks due Bhatnagar's First Alternative Formulation	Countries	Effect on Ranks due Bhatnagar's First Alternative Formulation
Mauritius	18up	Costa Rica	13down
Kuwait	17up	Botswana	11down
Qatar	13up	Algeria	10down
Brunei Darusallam	12up	Antigua& Barbuda	9down
UAE	10up	Fiji	8down
Saudi Arabia	10up	Dominica	8down

Table 4. Variations in ranks of prominently affected countries in HDR 1996 on replacing utility function from Atkinson based to Bhatnagar's First Alternative Formulation retaining corresponding threshold income levels in the HDRs

Countries	Effect on Ranks due to Bhatnagar's First Alternative Formulation	Countries	Effect on Ranks due to Bhatnagar's First Alternative Formulation
Qatar	15up	Costa Rica	14down
Kuwait	15up	Algeria	13down
UAE	13up	Iran Islamic Rep.	12down
St. Lucia	11up	Tunisia	11down
Mauritius	10up	Botswana	10down
Switzerland	10up	Uruguay	10down

Table 5. Variations in ranks of prominently affected countries in HDR 1995 on replacing utility function from Atkinson based to Bhatnagar's First Alternative Formulation retaining corresponding threshold income levels in the HDRs

Countries	Effect on Ranks due to Bhatnagar's First Alternative Formulation	Countries	Effect on Ranks due to Bhatnagar's First Alternative Formulation
Qatar	20up	Costa Rica	16down
St. Vincent	17up	Belize	14down
UAE	16up	Algeria	13down
Oman	14up	Tunisia	12down
Brunei Darussalam	14up	Botswana	11down
Mauritius	13up	Uruguay	9down

Table 6. Variations in ranks of prominently affected countries in HDR 1998 on replacing utility function from Atkinson based to Bhatnagar's Second Alternative Formulation retaining corresponding threshold income levels in the HDRs

Countries	Effect on Ranks due to Bhatnagar's Second Alternative Formulation	Countries	Effect on Ranks due to Bhatnagar's Second Alternative Formulation
Kuwait	14up	Costa Rica	9down
Qatar	13up	Botswana	8down
Mauritius	10up	Spain	7down
Brunei Darussalam	8up	St. Vincent	5down
Bahrain	8up	Panama	5down
UAE	7up	Hungary	5down

Table 7. Variations in ranks of prominently affected countries in HDR 1997 on replacing utility function from Atkinson based to Bhatnagar's Second Alternative Formulation retaining corresponding threshold income levels in the HDRs

Countries	Effect on Ranks due to Bhatnagar's Second Alternative Formulation	Countries	Effect on Ranks due to Bhatnagar's Second Alternative Formulation
Kuwait	12up	Costa Rica	9down
Mauritius	11up	Botswana	8down
Qatar	11up	Dominica	7down
Brunei Darussalam	10up	Uruguay	6down
UAE	7up	Spain	6down
Switzerland	7up	Colombia	6down

Table 8. Variations in ranks of prominently affected countries in HDR 1996 on replacing utility function from Atkinson based to Bhatnagar's Second Alternative Formulation retaining corresponding threshold income levels in the HDRs

Countries	Effect on Ranks due to Bhatnagar's Second Alternative Formulation	Countries	Effect on Ranks due to Bhatnagar's Second Alternative Formulation
UAE	11up	Costa Rica	11down
St. Lucia	11up	Algeria	8down
Qatar	11up	Uruguay	8down
Kuwait	10up	Botswana	7down
Switzerland	6up	Spain	6down
Surinam	6up	Iran Islamic Rep.	6down

Table 9. Variations in ranks of prominently affected countries in HDR 1995 on replacing utility function from Atkinson based to Bhatnagar's Second Alternative Formulation retaining corresponding threshold income levels in the HDRs

Countries	Effect on Ranks due to Bhatnagar's Second Alternative Formulation	Countries	Effect on Ranks due to Bhatnagar's Second Alternative Formulation
Qatar	17up	Costa Rica	14down
St. Vincent	17up	Belize	12down
UAE	15up	Uruguay	8down
Brunei Darussalam	12up	St. Kitts & Nevis	8down
Bahrain	11up	Iran Islamic Rep.	7down
Singapore	7up	Algeria	7down

Table 10. Number of affected countries in various HDRs on replacing utility function from Atkinson based with Bhatnagar's First & Second Alternative Formulations retaining corresponding threshold income levels in the HDRs

HDR description	Number of countries gaining Ranks at least by 5 positions due to Bhatnagar's First Alternative Formulation	Number of countries gaining Ranks at least by 5 positions due to Bhatnagar's Second Alternative Formulation	Number of countries losing Ranks at least by 5 positions due to Bhatnagar's First Alternative Formulation	Number of countries losing Ranks at least by 5 positions due to Bhatnagar's Second Alternative Formulation
(1)	(2)	(3)	(4)	(5)
HDR 1995	15	10	21	11
HDR 1996	17	10	17	12
HDR 1997	16	9	21	10
HDR 1998	12	7	21	10

Table 11. Countries in HDR 1998 affected at least by 5 ranks on replacing utility function from Atkinson based to Bhatnagar's First & Second Alternative Formulation retaining corresponding threshold income levels in the HDRs

Countries with Improved ranks exclusively due to Bhatnagar's First Alternative Formulation	Common list of Countries affected with Improved ranks due to Bhatnagar's First and Second Alternative Formulation	Countries with Declined ranks exclusively due to Bhatnagar's First Alternative Formulation	Common list of Countries affected with Declined ranks due to Bhatnagar's First and Second Alternative Formulation
(1)	(2)	(3)	(4)
St. Kitts & Nevis	Kuwait	Turkey	Costa Rica
Ukraine	Qatar	Uruguay	Botswana
Tajikistan	Mauritius	Algeria	Spain
Georgia	Brunei Darussalam	Argentina	St. Vincent
Cuba	Bahrain	Mexico	Panama
	UAE	Indonesia	Hungary
	Switzerland	Gabon	Fiji
		Grenada	Dominica
		Libyan Arab Jamah	Colombia
		Poland	Antigua & Barbuda
		Greece	

Table 12. Countries in HDR 1997 affected at least by 5 ranks on replacing utility function from Atkinson based to Bhatnagar's First & Second Alternative Formulation retaining corresponding threshold income levels in the HDRs

Countries with Improved ranks exclusively due to Bhatnagar's First Alternative Formulation	Common list of Countries affected with Improved ranks due to Bhatnagar's First and Second Alternative Formulation	Countries with Declined ranks exclusively due to Bhatnagar's First Alternative Formulation	Common list of Countries affected with Declined ranks due to Bhatnagar's First and Second Alternative Formulation
(1)	(2)	(3)	(4)
Malaysia	Kuwait	Iran Islamic Rep.	Costa Rica
Oman	Qatar	Syrian Arab Rep.	Botswana
Vietnam	Mauritius	Mongolia	Dominica
Armenia	Brunei Darussalam	Gabon	Uruguay
Georgia	UAE	Slovakia	Spain
Cuba	Switzerland	Greece	Colombia
St. Kitts & Nevis	Bahrain	Tunisia	Antigua & Barbuda
	Saudi Arabia	Panama	Hungary
	Luxembourg	Namibia	Fiji
		Chile	Algeria
		Brazil	

Table 13. Countries in HDR 1996 affected at least by 5 ranks on replacing utility function from Atkinson based to Bhatnagar's First & Second Alternative Formulation retaining corresponding threshold income levels in the HDRs

Countries with Improved ranks exclusively due to Bhatnagar's First Alternative Formulation	Common list of Countries affected with Improved ranks due to Bhatnagar's First and Second Alternative Formulation	Countries with Declined ranks exclusively due to Bhatnagar's First Alternative Formulation	Common list of Countries affected with Declined ranks due to Bhatnagar's First and Second Alternative Formulation
(1)	(2)	(3)	(4)
Oman	UAE	Syrian Arab Rep.	Costa Rica
Saudi Arabia	St. Lucia	Korea Rep.	Uruguay
Armenia	Qatar	Finland	Algeria
Luxembourg	Kuwait	Panama	Botswana
Cuba	Switzerland	Fiji	Spain
Grenada	Surinam		Iran Islamic Rep.
Samoe (Western)	Singapore		Greece
	Mauritius		Argentina
	Brunei Darussalam		Tunisia
	Bahrain		Slovakia
			Jordan
			Antigua & Barbuda

IJEDST

ISSN: 2321 – 7537
IJEDST (2014), 2(1):1-18

Table 14. Countries in HDR 1995 affected at least by 5 ranks on replacing utility function from Atkinson based to Bhatnagar's First & Second Alternative Formulation retaining corresponding threshold income levels in the HDRs

Countries with Improved ranks exclusively due to Bhatnagar's First Alternative Formulation	Common list of Countries affected with Improved ranks due to Bhatnagar's First and Second Alternative Formulation	Countries with Declined ranks exclusively due to Bhatnagar's First Alternative Formulation	Common list of Countries affected with Declined ranks due to Bhatnagar's First and Second Alternative Formulation
(1)	(2)	(3)	(4)
Saudi Arabia	St. Vincent	Syrian Arab Rep.	Costa Rica
Kuwait	Qatar	Pakistan	Belize
Armenia	UAE	Ukraine	Uruguay
Switzerland	Brunei Darussalam	Estonia	St. Kitts & Nevis
Venezuela	Bahrain	Poland	Iran Islamic Rep.
	Singapore	Morocco	Algeria
	Oman	Panama	Tunisia
	Moldova Rep.	Fiji	Spain
	Mauritius	Namibia	Botswana
	Libyan Arab Jamah	Brazil	Turkey
			Greece

Table 15. Variations in ranks of prominently affected countries in HDR 1998 on changing Bhatnagar's Second Alternative Formulation with Bhatnagar's First Alternative Formulation while retaining threshold income level intact

Countries	Effect on Ranks	Countries	Effect on Ranks
Mauritius	7up	Turkey	5down
Malaysia	7up	Botswana	4down
Tajikistan	5up	Antigua & Barbuda	4down
Kuwait	5up	Algeria	4down
St. Kitts & Nevis	4up	Argentina	4down
UAE	4up	Libyan Arab Jamah	4down

Table 16. Variations in ranks of prominently affected countries in HDR 1997 on changing Bhatnagar's Second Alternative Formulation with Bhatnagar's First Alternative Formulation while retaining threshold income level intact

Countries	Effect on Ranks	Countries	Effect on Ranks
Mauritius	7up	Namibia	5down
Saudi Arabia	7up	Algeria	5down
Kuwait	5up	Costa Rica	4down
Oman	4up	Tunisia	4down
Georgia	4up	Mongolia	4down
Azerbaijan	4up	Panama	3down

Table 17. Variations in ranks of prominently affected countries in HDR 1996 on changing Bhatnagar's Second Alternative Formulation with Bhatnagar's First Alternative Formulation while retaining threshold income level intact

Countries	Effect on Ranks	Countries	Effect on Ranks
Oman	6up	Tunisia	6down
Kuwait	5up	Iran Islamic Rep.	6down
Armenia	5up	Syrian Arab Rep.	5down
Mauritius	4up	Algeria	5down
Switzerland	4up	Turkey	4down
Qatar	4up	Cameroon	3down

Table 18. Variations in ranks of prominently affected countries in HDR 1995 on changing Bhatnagar's Second Alternative Formulation with Bhatnagar's First Alternative Formulation while retaining threshold income level intact

Countries	Effect on Ranks	Countries	Effect on Ranks
Oman	9up	Tunisia	6down
Saudi Arabia	8up	Algeria	6down
Mauritius	8up	Syrian Arab Rep.	5down
Libyan Arab Jamah	5up	Botswana	5down
Venezuela	4up	Poland	4down
Armenia	4up	Brazil	4down

Table 19. Most affected countries on applying the threshold income of HDR 1995 to fixed dataset of HDR 1998 while replacing Atkinson based formulation with Bhatnagar's First Alternative Formulation

Countries	Effect on Ranks	Countries	Effect on Ranks
Grenada	5up	Tajikistan	4down
Poland	3up	Switzerland	4down
Indonesia	3up	Mauritius	3down
Belarus	3up	Belize	3down
South Africa	3up	Georgia	3down
Czech Rep.	2up	Kuwait	3down

Table 20. Most affected countries on applying the threshold income of HDR 1996 to fixed dataset of HDR 1998 while replacing Atkinson based formulation with Bhatnagar's First Alternative Formulation

Countries	Effect on Ranks	Countries	Effect on Ranks
Grenada	2up	Mexico	2down
Turkey	2up	Brazil	1down
Tunisia	2up	Croatia	1down
Belarus	2up	Dominican Rep.	1down
Algeria	1up	Ecuador	1down
Antigua & Barbuda	1up	Georgia	1down

Table 21. Most affected countries on applying the threshold income of HDR 1997 to fixed dataset of HDR 1998 while replacing Atkinson based formulation with Bhatnagar's First Alternative Formulation

Countries	Effect on Ranks	Countries	Effect on Ranks
Uruguay	1up	Vietnam	1down
Turkey	1up	UAE	1down
Tunisia	1up	Ukraine	1down
South Africa	1up	Switzerland	1down
Maldives	1up	Sri Lanka	1down
Grenada	1up	Oman	1down

Table 22. Extent of Gain in ranks of most affected countries under Bhatnagar's First Alternative Formulation due to changing Threshold Income Level (TIL)

Country	On Using TIL-97	On Using TIL-96	On Using TIL-95
KUWAIT	1	1	3
MAURITIUS	1	1	3
MEXICO	1	2	2
OMAN	1	1	1
SRI LANKA	1	1	1
SWITZERLAND	1	1	4
UKRAINE	1	1	1
UNITED ARAB EMRATE	1	1	2
VIETNAM	1	1	1

Table 23. Extent of Decline in ranks of most affected countries under Bhatnagar's First Alternative Formulation due to changing Threshold Income Level (TIL)

Country	On Using TIL-97	On Using TIL-96	On Using TIL-95
ANTIGUA & BARBUDA	1	1	2
BELARUS	1	2	3
EGYPT	1	1	2
FINLAND	1	1	1
GRENADA	1	2	5
HONDURAS	1	1	1
MALDIVES	1	1	1
SLOVENIA	1	1	1
SOUTH AFRICA	1	1	3
TUNISIA	1	2	2
URUGUAY	1	1	1

Table 24. Most affected countries on applying the threshold income of HDR 1995 to fixed dataset of HDR 1998 under Bhatnagar's Second Alternative Formulation

Countries	Effect on Ranks	Countries	Effect on Ranks
Poland	6up	Saudi Arabia	7down
Grenada	6up	Oman	7down
Ecuador	6up	Libyan Arab Jamah.	5down
Russian Federation	5up	Brazil	4down
Lebanon	4up	Armenia	3down
Gabon	4up	Belize	3down

Table 25. Most affected countries on applying the threshold income of HDR 1996 to fixed dataset of HDR 1998 under Bhatnagar's Second Alternative Formulation

Countries	Effect on Ranks	Countries	Effect on Ranks
Lebanon	2up	Mexico	2down
Turkey	2up	Brazil	2down
Syrian Arab Rep.	2up	Cuba	2down
Senegal	2up	Hungary	2down
Poland	2up	Libyan Arab Jamah.	2down
Grenada	2up	Luxembourg	2down

Table 26. Most affected countries on applying the threshold income of HDR 1997 to fixed dataset of HDR 1998 under Bhatnagar's Second Alternative Formulation

Countries	Effect on Ranks	Countries	Effect on Ranks
Syrian Arab Rep.	2up	Mexico	2down
Lebanon	1up	Libyan Arab Jamah	2down
Belgium	1up	Cuba	2down
Bulgaria	1up	Uruguay	1down
Costa Rica	1up	Ukraine	1down
Grenada	1up	Uganda	1down

Table 27. Extent of Gain in ranks of most affected countries under Bhatnagar's Second Alternative Formulation due to changing Threshold Income Level (TIL)

Country	On Using TIL-97	On Using TIL-96	On Using TIL-95
CUBA	2	2	3
KUWAIT	1	1	1
LATVIA	1	1	1
LIBYAN ARAB JAMAH.	2	2	5
MALAWI	1	2	3
MEXICO	2	2	2
OMAN	1	1	7
PORTUGAL	1	1	1
SWEDEN	1	1	1
SWITZERLAND	1	1	2
TANZANIA U.R	1	1	1
UKRAINE	1	1	3
URUGUAY	1	1	1

Table 28. Extent of Decline in ranks of most affected countries under Bhatnagar's Second Alternative Formulation due to changing Threshold Income Level (TIL)

Country	On Using TIL-97	On using TIL-96	On Using TIL-95
ANGOLA	1	1	2
BELGIUM	1	1	1
BULGARIA	1	1	3
COSTA RICA	1	1	1
COTE D'LVOIRE	1	1	1
GRENADA	1	2	6
INDONESIA	1	1	2
KOREA REPUBLIC	1	1	1
LEBENON	1	2	4
NEW ZEALAND	1	1	1
PARAGUAY	1	1	1
POLAND	1	2	6
SENEGAL	1	2	3
UNITED ARAB EMRATE	1	1	1

IJEDST

ISSN: 2321 – 7537
IJEDST (2014), 2(1):1-18

Table 29. Effect on rankings of South Asian countries using Bhatnagar's First Alternative Formulation of multi-step utility function when threshold income level of HDR of one year is retained in the successive year

Country	New ranks using threshold income of HDR 1997		New ranks using threshold income of HDR 1996		New ranks using threshold income of HDR 1995	
	In HDR 1997	In HDR 1998	In HDR 1996	In HDR 1997	In HDR 1995	In HDR 1996
Col(1)	Col(2)	Col(3)	Col(4)	Col(5)	Col(6)	Col(7)
India	137	137	134	137	135	134
Pakistan	139	139	135	139	133	135
Nepal	154	153	152	154	151	152
Sri Lanka	88	86	86	88	93	85
Bangladesh	147	147	143	147	147	143
Maldives	110	98	106	110	115	106
Bhutan	156	157	159	156	160	159

Table 30. Most affected countries between HDR 1997 and HDR 1998 under Bhatnagar's First Alternative Formulation at threshold income level of HDR 1997

Countries	Variation in Ranks	Countries	Variation in Ranks
Turkmenistan	15down	Namibia	14up
Estonia	8down	Oman	13up
Saudi Arabia	7down	Maldives	12up
Azerbaijan	6down	Cambodia	12up
Mexico	6down	Brazil	7up
Yemen	5down	Romania	6up

Table 31. Most affected countries between HDR 1996 and HDR 1997 under Bhatnagar's First Alternative Formulation at threshold income level of HDR 1996

Countries	Variation in Ranks	Countries	Variation in Ranks
Latvia	32down	Syrian Arab Rep	17up
Botswana	27down	Grenada	16up
Rwanda	23down	Dominica	15up
Kazhakistan	20down	Turkey	12up
Iraq	16down	South Africa	9up
Ukraine	16down	Antigua &Barbuda	9up

Table 32. Most affected countries between HDR 1995 and HDR 1996 under Bhatnagar's First Alternative Formulation at threshold income level of HDR 1995

Countries	Variation in Ranks	Countries	Variation in Ranks
Korea DPR.	46down	Korea Rep.	48up
Belize	28down	Samoe (Western)	20up
Albania	20down	Romania	20up
Moldova Rep.	20down	St. Lucia	17up
Estonia	18down	Kuwait	17up
Ukraine	18down	Antigua &Barbuda	13up

Table 33. Most affected countries between HDR 1997 and HDR 1998 under Bhatnagar's Second Alternative Formulation at threshold income level of HDR 1997

Countries	Variation in Ranks	Countries	Variation in Ranks
Turkmenistan	17down	Namibia	15up
Estonia	8down	Maldives	13up
Mauritius	8down	Oman	12up
Azerbaijan	7down	Cambodia	12up
Georgia	7down	Brazil	11up
United Arab Emirate	7down	Algeria	7up

Table 34. Most affected countries between HDR 1996 and HDR 1997 under Bhatnagar's Second Alternative Formulation at threshold income level of HDR 1996

Countries	Variation in Ranks	Countries	Variation in Ranks
Latvia	34down	Syrian Arab Rep	19up
Botswana	24down	Grenada	17up
Rwanda	24down	Dominica	16up
Kazhakistan	22down	Turkey	11up
Iraq	16down	Antigua &Barbuda	11up
Ukraine	16down	South Africa	10up

Table 35. Most affected countries between HDR 1995 and HDR 1996 under Bhatnagar's Second Alternative Formulation at threshold income level of HDR 1995

Countries	Variation in Ranks	Countries	Variation in Ranks
Korea DPR.	48down	Korea Rep.	49up
Belize	25down	Romania	22up
Moldova Rep.	22down	St. Lucia	18up
Albania	21down	Samoe (Western)	17up
Estonia	19down	Antigua &Barbuda	17up
Ukraine	19down	Algeria	15up

Table 36. Ranks for South Asian countries due to Post-Atkinson based Formulation (PAF), Bhatnagar's First Alternative Formulation (BFAF) and Bhatnagar's Second Alternative Formulation (BSAF) for HDRs of 1995 & 1996

Country	Year 1995			Year 1996		
	PAF	BFAF	BSAF	PAF	BFAF	BSAF
(1)	(2)	(3)	(4)	(5)	(6)	(7)
India	133	135	134	134	134	134
Pakistan	135	133	131	135	135	135
Nepal	151	151	151	151	152	152
Sri Lanka	83	93	95	76	86	88
Bangladesh	147	147	148	141	143	143
Bhutan	160	160	160	159	159	159
Maldives	110	115	116	103	106	106

Table 37. Ranks for South Asian countries due to Post-Atkinson based Formulation (PAF), Bhatnagar's First Alternative Formulation (BFAF) and Bhatnagar's Second Alternative Formulation (BSAF) for HDRs of 1997 & 1998

Country	Year 1997			Year 1998		
	PAF	BFAF	BSAF	PAF	BFAF	BSAF
(1)	(2)	(3)	(4)	(5)	(6)	(7)
India	137	137	138	137	137	138
Pakistan	139	139	139	139	139	139
Nepal	154	154	154	152	153	153
Sri Lanka	77	88	94	77	86	88
Bangladesh	143	147	144	145	147	147
Bhutan	156	156	156	154	157	155
Maldives	107	110	110	99	98	97

Table 38. Ranks for some industrially advances countries due to Post-Atkinson based Formulation (PAF), Bhatnagar's First Alternative Formulation (BFAF) and Bhatnagar's Second Alternative Formulation (BSAF) for HDRs of 1995 & 1996

Country	Year 1995			Year 1996		
	PAF	BFAF	BSAF	PAF	BFAF	BSAF
(1)	(2)	(3)	(4)	(5)	(6)	(7)
USA	2	2	2	2	2	2
Japan	3	3	3	3	3	3
China	106	108	110	105	107	107
Germany	11	13	14	17	17	18
France	5	5	6	6	6	6
UK	17	17	18	16	16	17

Table 39. Ranks for some industrially advanced countries due to Post-Atkinson based Formulation (PAF), Bhatnagar's First Alternative Formulation (BFAF) and Bhatnagar's Second Alternative Formulation (BSAF) for HDRs of 1997 & 1998

Country	Year 1997			Year 1998		
	PAF	BFAF	BSAF	PAF	BFAF	BSAF
(1)	(2)	(3)	(4)	(5)	(6)	(7)
USA	2	2	3	2	2	2
Japan	5	5	6	5	5	6
China	104	109	108	105	106	106
Germany	18	17	19	18	18	19
France	3	3	2	4	4	3
UK	16	16	15	16	16	15

Old ranks in Table 1 are those which existed in the respective HDRs of UNDP, following the Atkinson based multi-step utility function, whereas new ranks corresponding to each year have been worked out by using the Bhatnagar's First and Second Alternative Formulations separately. The variations in the ranks for the countries in the Indian Sub-Continent have been found to be quite negligible by adopting any one of the two Alternatives. The corresponding scenarios in respect of other countries of the world covered in the UNDP's HDRs are discussed in the following sub-sections. The implications of using Bhatnagar's First

and Second Alternative Formulations would be examined in more depth in section 3.

CASE 1: IMPACT OF BHATNAGAR's FIRST ALTERNATIVE FORMULATION

By going beyond the horizon of Indian Sub-continent and applying Bhatnagar's First Alternative Formulation of multi-step utility function in HDR 1998, we observe that Kuwait, Qatar, Mauritius, UAE, Malaysia and Bahrain emerge as the most affected countries recording improvements in their ranks by at least 10 positions. In addition, other seven countries viz., Brunei Darussalam, Switzerland, St. Kitts & Nevis, Ukraine, Tajikistan, Georgia and Cuba also experience upward movements in the range of 5 to 9 ranks. On the other hand the countries like Costa Rica, Botswana, Spain, Antigua & Barbuda, Fiji and Dominica suffered in terms of their ranks declining the most due to replacement of the Atkinson based formulation with Bhatnagar's First Alternative Formulation in the HDR 1998. In addition to them, there exist as many as 15 other countries -4 belonging to the continent of North America, another 3 countries each from the continents of Africa, South America and Europe besides 2 more countries from Asia, which lose their ranks during the same period by 5 to 9 positions. The extent of variations for the countries which undergo the maximum upward or downward movements in their rank can be seen below.

While 16 countries covered under HDR 1997 are found to gain their ranks at least by 5 positions, as many as 21 another countries experience decline in their ranks by at least 5 positions on using Bhatnagar's First Alternative Formulation of multi-step utility function. Mauritius with upward movement in rank by 18 positions and Costa Rica losing rank by 13 positions are the most affected countries during the same period. Besides Mauritius, the other prominently affected countries undergoing improvement in their ranks include Kuwait, Qatar, Brunei Darussalam, UAE and Saudi Arabia. However, apart from Costa Rica, the other countries whose ranks are affected resulting fall in their ranks are Botswana, Algeria, Antigua & Barbados, Fiji and Dominica. The extent of variations for the countries which undergo the maximum upward or downward movements in their rank can be seen below.

In case when UNDP's Atkinson based multi-step utility function in the HDR 1996 is replaced with Bhatnagar's First Alternative Formulation, Qatar and Costa Rica emerge as the most affected countries. While 17 countries are affected in terms of gaining in their ranks by at least 5 positions, there are again another 17 countries experiencing decline in their ranks by at least 5 positions. Tally of 6 most affected countries whose ranks would go up on applying Bhatnagar's First Alternative Formulation as the utility function in the HDR 1996 includes besides Qatar other countries namely, Kuwait, UAE, St. Lucia, Mauritius and Switzerland. However, in addition to Costa Rica, the most affected countries whose ranks decline substantially are Algeria, Iran Islamic Rep, Tunisia, Botswana and Uruguay. The extent of variations for the countries which undergo the maximum upward or downward movements in their rank can be seen below.

Amongst the countries covered under HDR 1995 as many as 15 counties experience improvement in their ranks ranging between 5 to 20 while another 21 countries suffer decline in the ranks ranging between 5 to 16, as a result of applying Bhatnagar's First Alternative Formulation in place of the Atkinson based multi-step utility function. The most affected countries gaining in their ranks include Qatar, St. Vincent, UAE, Oman, Brunei Darussalam and Mauritius; on the other hand the countries losing their ranks the most are Costa Rica, Belize, Algeria, Tunisia, Botswana and Uruguay. In addition to them, out of remaining 9 countries whose ranks are improved, 4 belong to Asia, 3 to Europe and 1 each to Africa and South America. Likewise, out of the remaining 15 other countries experiencing decline in their ranks, 5 belong to Europe, 4 to Asia, 2 each belong to Africa and North America while 1 each to the continents of South America and Oceania. The extent of variations for the countries which undergo the maximum upward or downward movements in their rank can be seen below.

CASE 2: IMPACT OF BHATNAGAR's SECOND ALTERNATIVE FORMULATION

Let us now consider Bhatnagar's Second Alternative Formulation in lieu of the Bhatnagar's First Alternative Formulation as a replacement for the Atkinson based multi-step utility function in the HDRs from 1995 to 1998 in order to examine the variations in the ranking profile of various countries. The prominently affected countries in terms of either gaining or losing ranks significantly with reference to their UNDP's originally awarded ranks are presented in the Tables 6 to 9 given below.

BHATNAGAR's FIRST & SECOND ALTERNATIVE FORMULATIONS: FROM PERSPECTIVE OF THEIR IMPLICATIONS

From the mathematical formulations of the Bhatnagar's First and Second Alternative Formulations at Equations (1A) to (1F) and (2A) to (2I) respectively, though it is clear that the former specifies fragmentation of the entire range of real GDP per capita values in the multiples of twice of the threshold income level and the latter considers the segmentation of the range of income values exactly similar to UNDP's pattern in the multiples of the threshold level, but the precise distinction between their implications on the ranks of various countries covered in the HDRs from 1995 to 1998 is brought out as under.

(1) Since the width of intervals formed of the entire range real GDP per capita is smaller for Bhatnagar's Second Alternative Formulation as compared to that for Bhatnagar's First Alternative Formulation, the numbers of countries experiencing gain in their ranks or suffering from decline in their ranks with reference to UNDP's original ranks of the HDRs also turn out to be correspondingly smaller for the First Alternative as compared to the Second Formulation.

(2) Let $^{1995}I_1$, $^{1996}I_1$, $^{1997}I_1$ and $^{1998}I_1$ denote the four distinct sets of countries whose ranks have improved by upward movement say at least by 5 positions on applying Bhatnagar's First Alternative Formulation in place of the Atkinson based multi-step utility function during the HDR 1995, HDR 1996, HDR 1997 and HDR 1998 respectively. Similarly, let us denote by $^{1995}I_2$, $^{1996}I_2$, $^{1997}I_2$ and $^{1998}I_2$ the other four distinct sets of countries the ranks of whom have improved by upward movement by at least 5 positions on applying Bhatnagar's Second Alternative Formulation in place of the Atkinson based multi-step utility function during the HDR 1995, HDR 1996, HDR 1997 and HDR 1998 respectively. Though the rankings of the various countries as per Bhatnagar's First and Second Alternative Formulations are observed to be quite similar for some of them but not exactly same for all and in addition differ also in terms of magnitudes of their actual variation. However, the following relations of Set-theory are found to hold true:

$$^{1995}I_2 \cap {}^{1995}I_1 = {}^{1995}I_2$$
$$^{1996}I_2 \cap {}^{1996}I_1 = {}^{1996}I_2$$
$$^{1997}I_2 \cap {}^{1997}I_1 = {}^{1997}I_2$$
$$^{1998}I_2 \cap {}^{1998}I_1 = {}^{1998}I_2$$

Alternatively, we can say that set $^{1995}I_2$ is a subset of $^{1995}I_1$, set $^{1996}I_2$ is a subset of $^{1996}I_1$, set $^{1997}I_2$ is a subset of $^{1997}I_1$ and set $^{1998}I_2$ is a subset of $^{1998}I_1$. Equivalently, all the countries in any HDR which are affected on account of upward movement of their ranks under Bhatnagar's Second Alternative Formulation are also affected experiencing upward movement of ranks (though not by the same magnitude of variation) when Bhatnagar's First Alternative Formulation is applied to the same HDR data.

(3) Similar to the sets of the countries with improved ranks, let $^{1995}D_1$, $^{1996}D_1$, $^{1997}D_1$ and $^{1998}D_1$ denote the four distinct sets of countries the ranks of whom have declined by downward movement say at least by 5 positions on applying Bhatnagar's First Alternative Formulation in place of the Atkinson based multi-step utility function during the HDR 1995, HDR 1996, HDR 1997 and HDR 1998 respectively. Further corresponding to the Bhatnagar's Second Alternative Formulation, let us denote by $^{1995}D_2$, $^{1996}D_2$, $^{1997}D_2$ and $^{1998}D_2$ other four distinct sets of countries whose ranks have declined by downward movement by the same at least 5 positions on applying Bhatnagar's Second Alternative Formulation in place of the Atkinson based multi-step utility function during the HDR 1995, HDR 1996, HDR 1997 and HDR 1998 respectively. Though for some of the countries in any considered HDR the rankings as per Bhatnagar's First and Second Alternative Formulations are observed to be quite similar but the implications are not exactly same for all the countries covered under the same HDR. It may, however, be observed that even though there may be similarity in the descending order of variations of rank for some of the most affected countries but their actual magnitudes of variation are different. However, analogous to the sets of countries with improved ranks, the following relations of Set-theory are found to hold true in respect of those countries as well which suffer due to slippage from their originally awarded HDR-ranks:

$$^{1995}D_2 \cap {}^{1995}D_1 = {}^{1995}D_2$$
$$^{1996}D_2 \cap {}^{1996}D_1 = {}^{1996}D_2$$
$$^{1997}D_2 \cap {}^{1997}D_1 = {}^{1997}D_2$$
$$^{1998}D_2 \cap {}^{1998}D_1 = {}^{1998}D_2$$

Alternatively, we can say that set $^{1995}D_2$ is a subset of $^{1995}D_1$, set $^{1996}D_2$ is a subset of $^{1996}D_1$, set $^{1997}D_2$ is a subset of $^{1997}D_1$ and set $^{1998}D_2$ is a subset of $^{1998}D_1$. Equivalently, all the countries in any HDR which are affected on account of downward movement of their ranks under Bhatnagar's Second Alternative Formulation are also affected experiencing downward movement of ranks (though not by the same magnitude of variation) when Bhatnagar's First Alternative Formulation is applied to the same HDR data. On applying Bhatnagar's First and Second Alternative Formulations to the HDRs of 1995 to 1998, the exact number of countries affected through either upward movement or the downward movement in their ranks, with reference to the UNDP's original ranks in different HDRs can be seen below.

Evidently, the countries as per column (3) of the Table 10 are common with those in column (2) therein while the difference between these two columns specifies the countries which are exclusively affected with upward movement in the ranks by the Bhatnagar's First Alternative Formulation only. Similarly, the countries as per column (5) of the Table 10 are common with those in column (4) therein while the difference between these two columns specifies the countries which are exclusively affected with downward movement in the ranks by the Bhatnagar's First Alternative Formulation only. The precise lists of (a) countries gaining ranks at least by 5 positions exclusively under Bhatnagar's First Alternative Formulation, (b) common countries which evince improvement by at least 5 ranks by both Bhatnagar's First Alternative Formulation as well as Bhatnagar's Second Alternative Formulation, (c) countries losing ranks at least by 5 positions exclusively under Bhatnagar's First Alternative Formulation, (b) common countries which depict decline by at least 5 ranks by

both Bhatnagar's First Alternative Formulation as well as Bhatnagar's Second Alternative Formulation are presented for HDRs 1995 to 1998 in the following Tables from 11 to 14.

BHATNAGAR's FIRST & SECOND ALTERNATIVE FORMULATIONS: FROM STANCE OF MUTUAL IMPACT ON RANKING OF COUNTRIES

In the previous section, we glanced through the individual potential of Bhatnagar's First and Second Alternative Formulations vis-à-vis the Atkinson based multi-step utility function adopted by the UNDP in the HDRs from 1995 to 1998. Let us now compare and contrast these two Alternative Formulations mutually amongst themselves. To this end, we first compute the fresh HDI-ranks for all the countries while adopting the Bhatnagar's Second Alternative Formulation at the first instance and thereafter change the formulation to Bhatnagar's First Alternative Formulation and compute the revised ranks for them. Although both of them yield different rankings for most of the countries in any HDR, but the magnitude of variations on either side is not quite high. For as many as 52 countries in the HDR 1995, the variation between the ranks due to Bhatnagar's First and Second Alternative Formulation is observed to be nil. In the subsequent HDRs from 1996 to 1998, the numbers of such countries are found to be 68, 53 and 59 respectively.

We present in the following Tables 15 to 18 the extent of variations in the ranks of the most affected countries while comparing the two Alternative Formulations together.

Adopting Bhatnagar's First and Second Alternative Formulations for multi-step utility function, details of fresh HDI ranks for all countries in the HDRs from 1995 to 1998 have been separately enabled in **Appendix A-1**.

EFFECT ON THE RANKS OF COUNTRIES DUE TO REPLACEMENT OF UNDP's ATKINSON BASED MULTI-STEP UTILITY FUNCTION WITH BHATNAGAR's FIRST & SECOND ALTERNATIVE FORMULATIONS WHILE USING SAME HDR DATASET BUT VARYING THE LEVELS OF THRESHOLD INCOME

Considering the dataset of any HDR, we now allow an inter-play of the threshold income levels of other HDRs together with the change in the multi-step utility function from the Atkinson based formulation to Bhatnagar's First & Second Alternative Formulations respectively and study the resultant variations in the ranks of different countries. To take cognizance of the point at issue, we first adopt Bhatnagar's First Alternative Formulation for computing the utility from income and then observe the difference in the rankings of the countries by empirically considering the following sets of data:

(i) HDR dataset of 1998 retaining threshold income level of 1998 vis-a-vis the HDR dataset of 1998 with the threshold income level of 1997;
(ii) HDR dataset of 1998 retaining threshold income level of 1998 vis-a-vis the HDR dataset of 1998 with the threshold income level of 1996;
(iii) HDR dataset of 1998 retaining threshold income level of 1998 vis-a-vis the HDR dataset of 1998 with the threshold income level of 1995.

Tables 19 to 21 bring out the tally of most affected countries under Bhatnagar's First Alternative Formulation, in view of their original ranks of the HDR 1998 either having gone up prominently or slipped down drastically, on applying threshold income levels of HDR 1995, HDR 1996 and HDR 1997 respectively.

While applying the threshold income levels of HDRs of 1995, 1996 and 1997 to the HDR data of 1998 and computing fresh ranks for various countries under Bhatnagar's First Alternative Formulation of utility function, it is observed that the number of countries whose ranks in HDR 1998 using own threshold income level improved with reference to the application of threshold income levels of previous HDRs to the same HDR data of 1998 varies as directly proportional the time-lag. As many as 42 countries are found to have gained their ranks in the HDR 1998 vis-a-vis the scenario of utilizing threshold income level of HDR 1995. However, this number reduces to 24 and 14 when the improvement in ranks is considered in the HDR 1998 with reference to application of threshold income levels of HDR 1996 and HDR 1997 respectively. Similar trend is noticed in the case of countries losing their ranks in the HDR 1998 while comparing with situations of applying the threshold income levels of HDR 1995, HDR 1996 and HDR 1997. The corresponding number of such countries is seen as 41, 19 and 12 respectively. Tables 19 to 21 clearly depict the most affected countries. Whereas 85 countries are found having no change in their ranks while comparing their ranks of HDR 1998 with the ranks arrived at for them while using the threshold income level of HDR 1995 in the HDR data of 1998, this number climbs up to 122 and 141 respectively when the threshold income levels of HDR 1996 and HDR 1997 are applied.

While Tables 19 to 21 reveal the most affected countries whose ranks have either gone up or moved down under Bhatnagar's First Alternative Formulation on applying Threshold Income levels of the years 1995, 1996 and 1997 respectively to the HDR dataset of 1998, but as many as nine countries have uniformly performed in terms of their upward movement of ranks. Another eleven countries depicted the reverse performance in terms of their uniformly declining

ranks. Tables 22 and 23 indicate the gaining and losing countries respectively.

Let us now consider the case when instead of Bhatnagar's First Alternative Formulation, we decide to adopt Bhatnagar's Second Alternative Formulation for the purpose of discounting per capita GDP levels above threshold income level. On applying the threshold income levels of HDRs of 1995, 1996 and 1997 to the HDR data of 1998 and computing fresh ranks for various countries under Bhatnagar's Second Alternative Formulation of utility function, it is observed that the number of countries whose ranks in HDR 1998 using own threshold income level improved with reference to the application of threshold income levels of previous HDRs to the same HDR data of 1998 again varies directly with the time-lag. As many as 14 countries are found to have gained their ranks in the HDR 1998 vis-a-vis the scenario of utilizing threshold income level of HDR 1997. However, this number increases to 24 and 45 when the improvement in ranks is considered in the HDR 1998 with reference to application of threshold income levels of HDR 1996 and HDR 1995 respectively. Similar trend is noticed in the case of countries losing their ranks in the HDR 1998 while comparing with situations of applying the threshold income levels of HDR 1995, HDR 1996 and HDR 1997 to the dataset of HDR 1998. The corresponding number of such countries is seen as 41, 25 and 16 respectively. Whereas 85 countries are found having no change in their ranks while comparing their ranks of HDR 1998 with the ranks arrived at for them while using the threshold income level of HDR 1995 in the HDR data of 1998, this number climbs up to 122 and 141 respectively when the threshold income levels of HDR 1996 and HDR 1997 are applied. Adopting Bhatnagar's Second Alternative Formulation, the most affected countries whose original ranks of the HDR 1998 either go up or slip down, on applying threshold income levels of HDR 1995, HDR 1996 and HDR 1997 respectively, are presented in Tables 24 to 26.

The most affected countries whose ranks have gone up uniformly under Bhatnagar's Second Alternative Formulation on applying Threshold Income levels of the years 1995, 1996 and 1997 respectively to the HDR dataset of 1998 are presented in Tables 27. On the other hand Table 28 provides the details regarding the most affected countries in terms of uniform downward movement of ranks.

The values of the HDI for all the countries are though found to change in magnitude but there is no overall effect on the relative rankings for the countries which remain in the same original income-band as defined according to threshold income of HDR 1998 even when the threshold level of HDR 1998 is changed to the threshold income levels of previous HDRs of 1995, 1996 and 1997. This empirically drawn conclusion can be stated and proved mathematically as Corollary given below.

COROLLARY :

There would be no effect on the relative rankings of countries if they remain in the same income-band on the basis of threshold income level for any given HDR data even after replacing the threshold income level by any of its multiple under Bhatnagar's First Alternative Formulation of utility function.

PROOF:

Following Equation (1F) Bhatnagar's First Alternative Formulation in general for n^{th} income-band can be expressed as:

$$W(y) = \{\sum^{n}_{m=0} (1/2^{m})\, y^{*} \log(2m)\} + (1/2^{n})\, y^{*} \log (y/y^{*}), \text{ for } 2ny^{*} < y \leq 2(n+1)y^{*} \text{ and } n \geq 1 \quad (1)$$

Or that

$$W(y) = y^{*} \{a_n + b_n \log (y) - b_n \log (y^{*})\} \quad (2)$$

where $a_n = \sum^{n}_{m=0} (1/2^{m}) \log(2m)$ and $b_n = (1/2^{n})$.

Define a new threshold income level as a multiple of the original threshold income level as follows

$$y_{new}^{*} = ky^{*} \quad (3)$$

Using equation (2), the utility function for n^{th} income band, corresponding to new threshold level can be expressed as

$$W_{new}(y) = k\, W(y) - q_n \quad (4A)$$

Clearly $q_n = \{b_n\, k \log (k)\} y^{*}$ is a fixed constant within the given n^{th} income band. From equation (4A),

$$W_{new}(40000) = k\, W(40000) - q_n \quad (4B)$$

The index of adjusted GDP per capita corresponding to y_{new}^{*} is thus

$$\text{Adj GDP}_{new} = \{k\, W(y) - q_n - 100\} / \{k\, W(40000) - q_n - 100\} \quad (5A)$$

Or that

$$\text{Adj GDP}_{new} = \alpha_{1n}\, W(y) - \beta_{1n} \quad (5B)$$

where, $\alpha_{1n} = k/\{kW(40000) - q_n - 100\}$ and $\beta_{1n} = (q_n + 100)/\{kW(40000) - q_n - 100\}$ are fixed constants within the given n^{th} income band. Corresponding to the original threshold income level y^{*}, the index of adjusted GDP per capita can be written as

$$\text{Adj GDP}_{old} = \{W(y) - 100\} / \{W(40000) - 100\} \quad (6A)$$

Or that

$$\text{Adj GDP}_{old} = \alpha_{2n}\, W(y) - \beta_{2n} \quad (6B)$$

where, $\alpha_{2n} = 1/\{W(40000) - 100\}$ and $\beta_{2n} = 100/\{W(40000) - 100\}$ are again fixed constants within the n^{th} income band.

For any given HDR data, the life expectancy index and the educational attainment index remain independent of any change in the approach for income component. From (5A) and (6A), the change in the value of HDI

due to variation in the threshold income levels would, therefore, be

$$\delta_n (HDI) = \{(\alpha_{1n} - \alpha_{2n}) W(y) - (\beta_{1n} - \beta_{2n})\}/3 \quad (7)$$

Or that

$$\delta_n (HDI) = \alpha_n W(y) - \beta_n \quad (8)$$

where, $\alpha_n = (\alpha_{1n} - \alpha_{2n})/3$ and $\beta_n = (\beta_{1n} - \beta_{2n})/3$ are fixed constants for the given n^{th} income band. Clearly the values of HDI change by the same formula (8) for all the countries in n^{th} income-band on using a threshold income level which is a multiple (integer or fraction) of the given threshold income level. Hence the rankings of the countries with adjusted real GDP per capita levels falling in the given n^{th} income band would be affected by equal proportion, which means that the exact difference in their ranks would be zero. Since we have considered above any general n^{th} income band, it implies that the result will be true for all income bands individually.

EFFECT ON THE RANKS OF COUNTRIES DUE TO REPLACEMENT OF UNDP's ATKINSON BASED MULTI-STEP UTILITY FUNCTION WITH BHATNAGAR's FIRST & SECOND ALTERNATIVE FORMULATIONS WHILE APPLYING SAME THRESHOLD INCOME LEVEL TO HDR DATASET OF SUCCESSIVE YEARS

In this section, we study as to how the ranks of the countries vary when the threshold income level of one year is carried forward unaltered to the next year under Bhatnagar's First Alternative Formulation when applied to the HDR dataset of both the years. Taking cognizance of this scenario, we therefore consider the variations in the rankings of countries as evinced from the following sets of data:

(a) HDR data of 1997 using the threshold income level of 1997 vis-a-vis the HDR data of 1998 using the threshold income level of 1997, when Bhatnagar's First Alternative Formulation is adopted to both;

(b) HDR data of 1996 using the threshold income level of 1996 vis-a-vis the HDR data of 1997 using the threshold income level of 1996, when Bhatnagar's First Alternative Formulation is adopted to both;

(c) HDR data of 1995 using the threshold income level of 1995 vis-a-vis the HDR data of 1996 using the threshold income level of 1995, when Bhatnagar's First Alternative Formulation is adopted to both.

The magnitude of variations in the relative ranks of South Asian countries can be observed as follows:

The prominently affected countries in the World which have undergone major variations in the ranks in 1998, 1997 and 1996 using the threshold income levels of HDR 1997, HDR 1996 and HDR 1995 respectively are considered as follows.

As many as 26 countries gain by 5 or more positions on the basis of data of HDR 1996 as compared to their ranks of HDR 1995 when the threshold level of income of 1995 is replicated in 1996 in place of the actual threshold level while using Bhatnagar's First Alternative Formulation; however, the number of correspondingly losing countries stands as 19. Similarly, the number of countries which gain at least by 5 positions in their ranks on retaining the threshold income level of the HDR 1997 also in the successive HDR 1998 and deploying Bhatnagar's First Alternative Formulation turns out to be 7 as against the corresponding number of losing countries standing at 10. It is further observed that on the similar lines, while 17 countries emerge as the ones having recorded gain in their ranks by at-least 5 positions, there are as many as 43 countries on the other hand which lose their ranks by 5 positions or more, when the threshold levels of income of 1997 is replicated also in the HDR data of 1998 under Bhatnagar's First Alternative Formulation. Amongst the above-specified number of countries, the most affected countries which have emerged out prominently as either gainers in ranks or as the losers when the threshold income levels of the HDRs of 1997, 1996 and 1995 are replicated in successive HDR of 1998, 1997 and 1996 respectively and comparisons are made between consecutive years namely, 1997 vis-à-vis 1998, 1996 vis-à-vis 1997 and 1995 vis-à-vis 1996 adopting Bhatnagar's First Alternative Formulation are presented in the Tables 30 to 32.

In lieu of Bhatnagar's First Alternative Formulation, if we now adopt Bhatnagar's Second Alternative Formulation of the utility function, it is found that for 09, 19 and 27 numbers of countries in HDR 1998, 1997 and 1996 the ranks are improved at least by 5 positions when computed at the threshold income levels of just preceding years namely, 1997, 1996 and 1995 respectively; while corresponding decline in the same three years is noticed for 13, 46 and 24 countries respectively. Further, both the approaches for formulation of multi-step utility function are found to yield almost identical set of prominently affected countries, although the extents of variation in their ranks are different. We present below in Tables 33 to 35 the most affected countries which have emerged out prominently as either gainers in ranks or as the losers under Bhatnagar's Second Alternative Formulation when the threshold income levels of the HDRs of 1997, 1996 and 1995 are replicated in successive HDR of 1998, 1997 and 1996 respectively and comparisons are made between consecutive years namely, 1997 vis-à-vis 1998, 1996 vis-à-vis 1997 and 1995 vis-à-vis 1996.

EFFECT ON THE RANKS OF COUNTRIES WHEN BHATNAGAR's FIRST & SECOND ALTERNATIVE FORMULATIONS OF UTILITY FUNCTION ARE COMPARED WITH UNDP's POST-1998 APPROACH

UNDP shifted to adoption of logarithmic function of the income as its transformed approach in HDRs for formulation of utility function with effect from 1999 but did not reveal the scenario of the relative rankings of the countries on replacement of Atkinson based formulation with a non-fragmented logarithmic utility function in HDR. We now consider Bhatnagar's First Alternative Formulation and Bhatnagar's Second Alternative Formulation with a view to study their impacts on the ranks of South Asian countries in the Indian sub-continent as compared to the post-1998's transformed approach (being termed here as Post-Atkinson based approach) of UNDP. In Tables 36 and 37, the column (2) and (5) give the ranks which have been computed by us adopting the Post-Atkinson based approach of UNDP to the HDR data of 1995 & 1996 and 1997 & 1998 respectively. The corresponding ranks for these countries under Bhatnagar's First Alternative Formulation and Bhatnagar's Second Alternative Formulation are also presented in the Table 36 and 37.

Amongst the South Asian Countries, the two formulations yield identical ranks for Nepal in all the four years while for India and Pakistan the only exception was in the year 1995. Minor variations in the relative ranks occur in the case of remaining countries. The variations in the ranks of some of the industrially advanced countries of the World are presented in the Table 38 and 39.

It follows that by and large both the alternative multi-step formulation of the utility function viz., Bhatnagar's First Alternative Formulation and Bhatnagar's Second Alternative Formulations yield almost identical rankings vis-à-vis UNDP's Post-Atkinson based approach in the HDR for industrially advanced countries also, similar to the findings for South Asian countries in the Indian sub-continent.

REFERENCES

[1] Bhatnagar Ravi Kant (2001) An Analysis of the Evolution of the Human Development Index with special reference to its Income Component, **Bangladesh Development Studies**, Vol. XXVII, No.3, Pp. 35-65.

[2] Bhatnagar Ravi Kant (2002) Problems with UNDP Multi-step Utility Function and Their Resolutions: Generalization of Some Results, **Indian Journal of Economics**, Vol. LXXXIII, No. 328, Pp. 123-138.

Appendix A-1

Effect on ranks of Countries due to Bhatnagar's First Alternative Formulation vis-à-vis Bhatnagar's Second Alternative Formulation of utility function during 1995 to 1998

country	1995-BFAF	1995-BSAF	1996-BFAF	1996-BSAF	1997-BFAF	1997-BSAF	1998-BFAF	1998-BSAF
ALBANIA	82	82	102	103	101	101	104	104
ALGERIA	98	92	82	77	92	87	89	85
ANGOLA	164	164	165	165	159	158	160	159
ANTIGUA & BARBUDA	58	56	46	45	38	35	38	34
ARGENTINA	34	32	38	36	40	38	42	38
ARMENIA	83	87	87	92	98	99	95	95
AUSTRIA	14	13	13	13	11	12	11	13
AZERBAIJAN	95	98	94	95	102	106	108	109
BAHAMAS	25	25	25	25	28	29	29	30
BAHRAIN	31	33	32	34	33	36	33	35
BANGLADESH	147	148	143	143	147	144	147	147
BARBADOS	28	26	27	26	29	27	28	28
BELARUS	46	46	60	60	62	62	68	68
BELGIUM	12	12	10	11	10	11	10	12
BELIZE	43	41	71	68	66	63	63	62
BENIN	159	158	157	157	149	148	146	146
BHUTAN	160	160	159	159	156	156	157	155
BOLIVIA	111	112	110	110	114	114	115	116
BOTSWANA	85	80	81	78	108	105	109	105
BRAZIL	70	66	61	61	74	71	66	63
BRUNEI DARUSSALAM	27	29	28	30	26	28	26	27
BULGARIA	67	65	63	63	70	69	65	67

Effect on ranks of Countries due to Bhatnagar's First Alternative Formulation vis-à-vis Bhatnagar's Second Alternative Formulation of utility function during 1995 to 1998

country	1995-BFAF	1995-BSAF	1996-BFAF	1996-BSAF	1997-BFAF	1997-BSAF	1998-BFAF	1998-BSAF
dBURUNDI	165	165	166	166	170	169	170	170
CAMBODIA	154	153	155	156	152	153	140	140
CAMEROON	127	127	130	127	136	134	133	132
CANADA	1	1	1	1	1	1	1	1
CAPE VERDE	120	122	121	121	120	122	118	117
CENTRAL AFRICAN R.	148	149	147	147	151	150	154	154
CHAD	161	161	161	162	164	164	163	163
CHILE	37	35	37	35	37	34	34	33
CHINA	108	110	107	107	109	108	106	106
COLOMBIA	60	60	53	53	57	57	59	58
CONGO	124	123	125	125	130	130	128	128
COSTA RICA	44	42	45	42	46	42	47	43
COTE D'LVOIRE	149	147	149	149	148	147	150	149
CUBA	71	73	73	76	80	83	80	81
CYPRUS	23	23	24	24	27	26	27	25
CZECH REP.	39	38	40	38	39	39	39	37
DENMARK	15	16	14	14	15	16	14	16
DJIBOUTI	157	157	163	164	162	162	162	162
DOMINICA	69	69	64	64	49	48	49	46
DOMINICAN REP.	97	97	91	89	91	89	88	89
ECUADOR	72	70	66	66	72	72	75	75
EGYPT	110	109	108	108	113	112	116	114
EL SALVADOR	113	114	114	114	112	113	112	113
ESTONIA	48	47	67	67	69	70	77	77
ETHOPIA	169	169	168	168	169	170	169	169
FIJI	52	50	52	50	54	51	52	49
FINLAND	9	7	11	8	9	8	9	8
FRANCE	5	6	6	6	3	2	4	3
GABON	118	117	123	122	126	123	125	124
GAMBIA	163	162	164	163	165	165	165	165
GEORGIA	88	91	100	100	100	104	103	107
GERMANY	13	14	17	18	17	19	18	19
GHANA	130	130	132	131	134	133	134	134
GREECE	30	27	30	27	25	24	25	24
GRENADA	68	67	72	74	56	55	56	54
GUINEA	168	168	162	160	168	167	168	167
GUINEA BISSAU	162	163	160	161	163	163	164	164
EQATORIAL GUINEA	140	141	133	133	135	136	136	135
GUYANA	104	105	101	102	104	103	101	100
HAITI	146	146	144	145	155	155	155	157
HONG KONG	20	20	19	21	21	21	21	21
HONDURAS	112	113	112	113	115	115	120	119
HUNGARY	49	48	50	49	55	53	53	52
ICELAND	8	8	8	7	6	5	6	5
INDIA	135	134	134	134	137	138	137	138
INDONESIA	105	104	105	104	103	100	102	99
IRAQ	109	107	111	109	127	127	127	127
IRELAND	21	21	21	19	18	18	17	17
ISRAEL	22	22	23	22	23	23	23	23
ITALY	18	19	20	20	20	20	19	20
JAMAICA	90	88	85	87	81	81	81	82
JAPAN	3	3	3	3	5	6	5	6
KAZAKHISTAN	66	64	70	70	90	92	94	93
KENYA	128	128	127	128	132	132	135	136
KOREA D.P.R	32	31	80	82	71	73	71	72

Effect on ranks of Countries due to Bhatnagar's First Alternative Formulation vis-à-vis Bhatnagar's Second Alternative Formulation of utility function during 1995 to 1998

country	1995-BFAF	1995-BSAF	1996-BFAF	1996-BSAF	1997-BFAF	1997-BSAF	1998-BFAF	1998-BSAF
KOREA REPUBLIC	80	83	34	32	32	32	32	32
KUWAIT	55	58	36	41	36	41	35	40
KYRGYZTAN	89	90	95	96	105	107	107	108
LAO PEOPLE'S D.R	138	137	139	139	138	137	138	137
LATVIA	51	49	57	56	89	91	90	91
LESOTHO	129	129	129	129	133	135	132	133
LIBYAN ARAB JAMAH.	63	68	62	62	68	67	69	65
LUXEMBOURG	24	24	22	23	22	22	22	22
MADAGASKAR	134	135	146	148	150	151	151	152
MALAWI	153	154	154	154	158	160	158	158
MALDIVES	115	116	106	106	110	110	98	97
MALI	172	172	170	170	171	171	171	171
MALTA	38	36	31	29	31	31	30	29
MAURITANIA	150	150	150	150	153	152	152	151
MAURITIUS	47	55	44	48	43	50	44	51
MEXICO	50	53	49	51	50	52	55	53
MOLDOVA REP.	76	76	96	98	107	109	110	111
MONGOLIA	106	108	109	111	106	102	105	103
MOROCCO	122	120	124	124	123	121	126	126
MOZAMBIQUE	166	166	167	167	166	166	166	166
MYANMAR	131	132	131	132	131	131	131	131
NAMIBIA	114	111	120	119	125	120	111	110
NEPAL	151	151	152	152	154	154	153	153
NETHERLAND	4	4	7	5	7	7	7	7
NEW ZEALAND	19	17	15	15	13	10	13	10
NICARAGUA	107	106	115	115	124	126	124	125
NIGER	174	174	174	174	173	173	173	173
NIGERIA	142	142	138	138	142	141	143	143
NORWAY	6	5	4	4	4	4	3	4
OMAN	77	86	75	81	82	86	67	70
PAKISTAN	133	131	135	135	139	139	139	139
PANAMA	54	51	48	47	52	49	51	50
PAPUA NEW GUINEA	125	126	126	126	128	128	130	130
PARAGUAY	91	89	84	85	93	94	91	92
PERU	96	94	92	91	87	88	83	86
PHILLIPINES	100	100	97	97	97	97	96	96
POLAND	56	52	55	55	59	58	57	55
PORTUGAL	33	34	33	33	30	30	31	31
QATAR	36	39	35	39	42	44	40	44
ROMANIA	94	96	74	73	79	79	73	74
RUSSIAN FEDERATION	53	54	58	57	67	66	72	73
SAINT KITTS & NEVIS	45	45	41	43	44	46	43	47
SAINT LUCIA	81	84	65	65	60	60	61	61
SAINT VINCENT	62	62	69	71	61	61	60	60
SAO TOME & PRINCIPE	132	133	128	130	122	125	121	121
SAUDI ARABIA	64	72	56	59	63	68	70	69
SENEGAL	156	155	156	155	161	161	161	161
SEYCHELLES	65	63	59	58	51	54	54	56
SIERRA LEONE	173	173	173	173	175	174	174	174
SINGAPORE	26	28	26	28	24	25	24	26
SLOVAKIA	42	43	47	46	48	45	46	45
SOUTH AFRICA	99	99	103	101	94	93	93	90
SPAIN	16	15	18	16	19	17	20	18
SRI LANKA	93	95	86	88	88	90	86	88
SUDAN	145	144	148	146	157	157	156	156

Effect on ranks of Countries due to Bhatnagar's First Alternative Formulation vis-à-vis Bhatnagar's Second Alternative Formulation of utility function during 1995 to 1998

country	1995-BFAF	1995-BSAF	1996-BFAF	1996-BSAF	1997-BFAF	1997-BSAF	1998-BFAF	1998-BSAF
SURINAM	75	75	68	69	64	65	62	64
SWAZILAND	123	124	113	112	117	116	117	115
SWEDEN	10	9	9	10	12	13	12	9
SWITZERLAND	7	10	5	9	8	9	8	11
SYRIAN ARAB REP.	84	79	99	94	83	80	84	83
TAJIKISTAN	103	103	104	105	111	111	113	118
TANZANIA U.R	144	145	142	142	146	149	148	148
THAILAND	61	61	54	54	58	59	58	59
TOGO	139	140	140	140	145	145	144	144
TRINIDAD & TOBAGO	35	37	39	37	41	40	41	39
TUNISIA	87	81	89	83	86	82	87	84
TURMENISTAN	86	85	90	90	84	84	99	101
TURKEY	74	71	88	84	76	77	76	71
UNITED ARAB EMRATE	29	30	29	31	34	37	37	41
UGANDA	155	156	153	153	160	159	159	160
UNITED KINGDOM	17	18	16	17	16	15	16	15
UKRAINE	59	57	79	80	95	95	97	98
URUGUAY	41	40	42	40	45	43	45	42
USA	2	2	2	2	2	3	2	2
UZBEKISTAN	92	93	93	93	99	98	100	102
VANATU	119	119	119	118	121	124	123	123
VENEZUELA	40	44	43	44	47	47	48	48
VIETNAM	117	118	117	120	116	117	119	120
YEMEN	141	139	145	144	144	146	149	150
ZAMBIA	136	136	136	136	143	143	145	145
ZIMBABWE	121	121	122	123	129	129	129	129

Legends: BFAF: stands for Bhatnagar's First Alternative Formulation
BSAF: stands for Bhatnagar's Second Alternative Formulation